A simple model for wealth-decency

(Draft)

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1 Main

Consider the society as a set of people gathered around each other who know everything about each other's wealth. We denote the set by $\mathcal{X}$ and a member of society by x. The more wealth one has, the more fun he gets, (*fun* differs from *utility* to be introduced shortly.). We denote the fun as a function of wealth[1], w, as $f(w)$. Without loss of generality[2] we may assume

$$f(0) = 0$$

$$\frac{df}{dw}\bigg|_{w=0} = 1$$

It is also reasonable to assume[3]

$$\frac{d^2 f}{dw^2} \leq 0$$

Finally assume that there are people in society with all possible amounts of wealth. Mathematically

$$\forall w \geq 0 \exists x \in \mathcal{X} \quad s.t. \quad W(x) = w$$

How happy is a member of society with wealth w? He sure enjoys the fun he is getting out of his money, but he would also like (hopefully) other people to enjoy their lives! The simplest way to express his *utility* (or *joy*) will therefore be

$$U(x) = f(W(x)) + D(x) \sum_{x' \in \mathcal{X}} f(W(x'))$$

With $D(x)$ measuring how *descent* x is.

Now consider some $x' \in \mathcal{X}$. Will x give x' some of his money? The answer is affirmative only when x's joy will increase by donating some small money δ[4].

$$\Delta U(x) = \delta[D(x)f'(W(x')) - (1 + D(x))f'(W(x))]$$

This means he will keep donating money until there is no joy in it.

$$\forall x' \quad D(x)f'(W(x')) \leq (1 + D(x))f'(W(x))$$

Which implies

$$f'(W(x)) = \frac{D(x)}{1 + D(x)}$$

or

$$D(x) = \frac{f'(W(x))}{f'(W(x)) - 1}$$

The concavity of f simply states that the wealthier x is, the less decent he will be! In fact you may have started without assuming the concavity of f and proved that otherwise the society will have no stable equilibrium.[5]

2 What did we neglect? or "you can't be serious!"

Well, we neglected many things. First of all, people have incomes and don't become poorer by donating money and perhaps more importantly donating money to the poorer is perhaps not the best way to fight poverty. It is not also the case that donating money has no cost, people are perhaps a little lazy in doing so. To be revised...

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[1]Which is itself a function of x, namely $W(x)$

[2]Except for the case $f(0) = -\infty$!

[3]When you've got nothing in your pockets, a dollar means much more to you than when you are *Bill Gates rich*!

[4]There is no question that x' will enjoy this act.

[5]In fact the situation will be very similar to the case of systems with negative heat capacities!