PSU Physics PhD Qualifying Exam Solutions
Fall 2005

Koorosh Sadri

May 6, 2022

Candidacy Exam
Department of Physics
August 20, 2005

Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants:

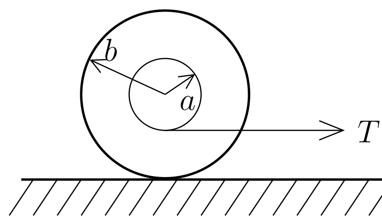
Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Gas constant	R	$8.314\,\mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A}^{-2}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5}\,\mathrm{N\,m}^{-2}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m}^{-2}\,\mathrm{K}^{-4}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$
Origin of temperature scales	$0\,^\circ\mathrm{C} = 273\,\mathrm{K}$	

I–1. A reel consists of a cylindrical hub of radius a and two circular end pieces of radius b. The mass of the complete reel is m and its moment of inertia about its long axis is I. The reel rests on horizontal table. The end of a light string is attached to the hub and wrapped around it, and a tension T in a horizontal direction is applied to the free end of the string, as shown in the figure. The coefficient of friction is large enough that the reel rolls on the table.

Determine:

(a) the frictional force exerted by the table,

(b) the direction in which the reel begins to move.



I–2. Two argon atoms are located along the x-axis at $x = \pm a$. Each has an isotropic polarizability α, such that the atom develops a dipole moment $\mathbf{p} = \alpha \mathbf{E}_l$ when subject to a local electric field $\mathbf{E}_l$. Obtain the total dipole moment of the two-atom system under each of the following conditions:

(a) When a constant electric field of size E_0 is imposed along the x-axis. ($\mathbf{E} = \hat{\mathbf{x}} E_0$)

(b) When a constant electric field of size E_0 is imposed along the y-axis. ($\mathbf{E} = \hat{\mathbf{y}} E_0$)

I–3. According to simple kinetic theory, the thermal conductivity of a gas is given by the expression

$$K = \frac{1}{3} C_V \lambda \langle v \rangle \, , \tag{I–1}$$

where C_V is the heat capacity of the gas at constant volume, per unit volume, λ is the mean distance between collisions, and $\langle v \rangle$ is the mean speed of the molecules. Approximate all molecules as spherical, and until part (d) ignore their internal vibrations, and treat the remaining degrees of freedom as classical.

(a) Find λ in terms of the number density of the gas and the radius of the molecules.

(b) How do the quantities on the right-hand-side of Eq. (I–1) depend on temperature T? Deduce the temperature dependence of K.

(c) Regard methane gas as a collection of spherical molecules 1.7 times the radius of argon atoms. The atomic weight of argon is 40, and the molecular weight of methane is 16. Estimate the ratio of the thermal conductivity of methane to argon gas, given that the gases have the same number density. [Hint: The molar heat capacity at constant volume, c_V, is $Rf/2$ for a gas with f degrees of freedom.]

(d) Would we expect this ratio to increase or decrease if the methane molecules were vibrationally excited by collisions.

I–4. Explain the significance of the photoelectric effect for the discovery of quantum mechanics.

In a photoelectric experiment monochromatic light of wavelength λ falls on a potassium surface. It is found that the stopping potential is 1.91 V for $\lambda = 3000\,\text{Å}$, and 0.88 V for $\lambda = 4000\,\text{Å}$. From these data calculate

(a) A value for Planck's constant given the values for the size of the charge of the electron e and for the speed of light c that are given in the table at the front of this exam.

(b) The work function W for potassium.

(c) The threshold frequency ν_{t} for potassium.

Candidacy Exam
Department of Physics
August 20, 2005

Part II

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants:

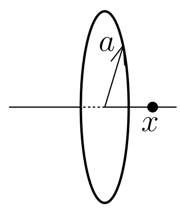
Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Gas constant	R	$8.314\,\mathrm{J\,mol}^{-1}\,\mathrm{K}^{-1}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A}^{-2}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5}\,\mathrm{N\,m}^{-2}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m}^{-2}\,\mathrm{K}^{-4}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$
Origin of temperature scales	$0\,^\circ\mathrm{C} = 273\,\mathrm{K}$	

II–1. The muon is a particle with mass $m_\mu = 106\,\mathrm{MeV}/c^2$ and mean lifetime $\tau_\mu = 2.20\,\mu\mathrm{s}$. A beam is made of muons of energy 100 GeV.

(a) By what fraction does the speed of the muons deviate from the speed of light c?

(b) What is the distance the beam travels before a fraction $1/e$ of the muons have decayed?

II–2. An electron is placed at the center of a uniform thin ring of charge q and radius a. The mass of the electron is m. It is then displaced a small distance x along the axis of the ring ($x \ll a$). Derive a formula for the frequency of small oscillations.



II–3. An air bubble of $20\,\mathrm{cm}^3$ volume is at the bottom of a lake 40 m deep where the temperature is 4.0 °C. The bubble rises to the surface, which is at a temperature of 20 °C. Take the temperature of the bubble to be the same as that of the surface water and find its volume just before it reaches the surface.

II–4. Consider a particle of mass m constrained to move in one dimension, along the x-axis. The particle experiences a harmonic oscillator potential $V(x) = \frac{1}{2}kx^2$. A natural angular frequency for the oscillator is defined by $\omega = \sqrt{k/m}$.

The first 6 normalized wave functions (energy eigenfunctions) are shown in Fig. II–1 on page II–3. Note that the wave functions are expressed as functions of the dimensionless variable $y \equiv x\sqrt{m\omega/\hbar}$ and are normalized so that $\int_{-\infty}^{\infty} \psi_n^*(y)\psi_n(y)\, dy = 1$.

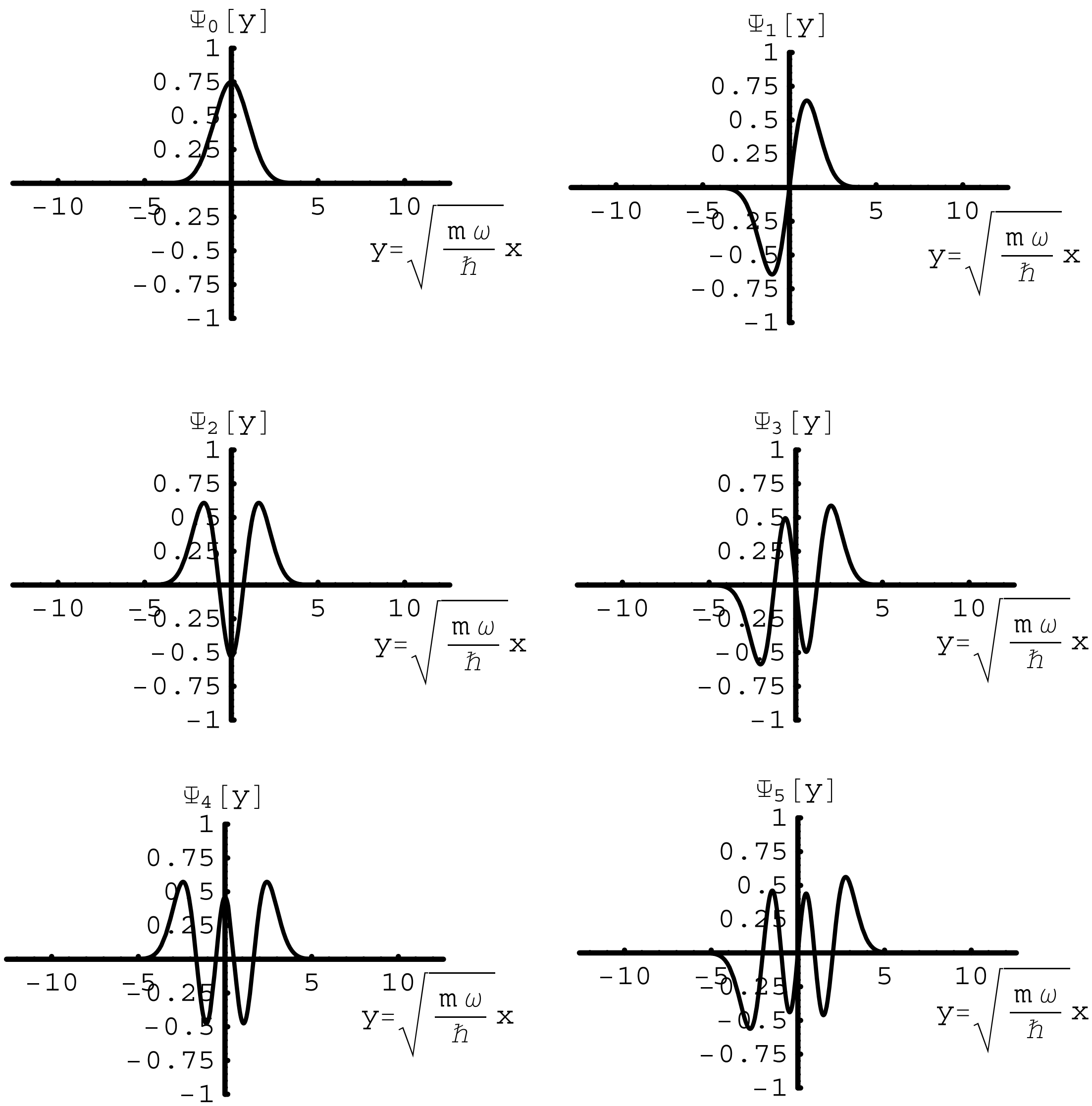
(a) Write down the energies associated with each of the first 6 wave functions.

(b) Now assume that the potential $V(x)$ is modified to be infinite for $x < 0$,

$$V_1(x) = \begin{cases} \frac{1}{2}kx^2 & \text{for } x \geq 0, \\ \infty & \text{for } x < 0. \end{cases} \tag{II–1}$$

Determine the three lowest energy eigenvalues for the system when the potential is $V_1(x)$.

(c) Assume the system initially has potential $V_1(x)$ and is in the ground state for that potential. The potential then suddenly changes to $V(x)$. Calculate the probability that the system is in the new ground state after the sudden change in potential. *Note: Take careful note of the details of the normalization condition that is applied to the wave functions.*



$$\Psi_0(y) = e^{-y^2/2}\,\frac{1}{\pi^{1/4}}, \qquad \Psi_1(y) = e^{-y^2/2}\,\frac{\sqrt{2}\,y}{\pi^{1/4}},$$

$$\Psi_2(y) = e^{-y^2/2}\,\frac{-1+2y^2}{\sqrt{2}\,\pi^{1/4}}, \qquad \Psi_3(y) = e^{-y^2/2}\,\frac{-3y+2y^3}{\sqrt{3}\,\pi^{1/4}}$$

$$\Psi_4(y) = e^{-y^2/2}\,\frac{3-12y^2+4y^4}{2\sqrt{6}\,\pi^{1/4}} \qquad \Psi_5(y) = e^{-y^2/2}\,\frac{15y-20y^3+4y^5}{2\sqrt{15}\,\pi^{1/4}}$$

Figure II–1: Plots of wave functions for Problem II–4.

I.1

a, b) Assuming an unknown friction force, f leftwards and an unknown angular acceleration α rightwards, the equations of motion are

$$\begin{cases} mb\alpha = T - f \\ I\alpha = fb - Ta \end{cases}$$

which solve as

$$\boxed{f = \frac{I + mab}{I + mb^2}T} \; ; \; \boxed{\alpha = \frac{b - a}{I + mb^2}T > 0}$$

I.2

a) Initially, the dipoles will be in the x direction. Since the secondary fields wil also be in the x direction, we are safe to assume that the dipole moments for both atoms are in the form $\mathbf{p} = \alpha E_l \hat{\mathbf{x}}$. Then, using the potential formula

$$\phi = \frac{\mathbf{p}.\boldsymbol{\mathscr{r}}}{4\pi\varepsilon_0 \mathscr{r}^3}$$

we can write E_l in terms of itself as

$$E_l = E_0 + \frac{\alpha E_l}{16\pi\varepsilon_0 a^3}$$

which gives

$$E_l = \frac{E_0}{1 - \frac{\alpha}{16\pi\varepsilon_0 a^3}}$$

Finally, the total dipole moment is found as

$$\mathbf{p}_{tot.} = 2\alpha \mathbf{E}_l = \frac{2\alpha}{1 - \frac{\alpha}{16\pi\varepsilon_0 a^3}} \mathbf{E}_0$$

Equivalently

$$\boxed{\alpha^x_{(eq.)} = \frac{2\alpha}{1 - \frac{\alpha}{16\pi\varepsilon_0 a^3}}}$$

b) A similar procedure leads to the answer

$$\boxed{\alpha^y_{(eq.)} = \frac{2\alpha}{1 + \frac{\alpha}{32\pi\varepsilon_0 a^3}}}$$

I.3

a) After a time t, the probability that an atom moving with speed $\mathbf{v}$, *does not* collide with another atom of velocity $\mathbf{u}$ (with some tolerance box $d\mathbf{u}$), is given by

$$\exp\big(- 4\pi a^2 \, |\mathbf{v} - \mathbf{u}| t \, n(\mathbf{u}) d\mathbf{u}\big)^1$$

Therefore, the probability that it survives all possible collisions after some time t, is given by

$$\exp\Big[- 4\pi a^2 nt \, \mathbf{E}_{\mathbf{U}}\big(|\mathbf{v} - \mathbf{U}|\big)\Big]$$

[1] I have assumed $a \ll \lambda$ in computing the swept volume.

In other words, the mean free time for this atom is

$$\tau(\mathbf{v}) = \frac{1}{4\pi a^2 n\, \mathbf{E}_{\mathbf{U}}\big(|\mathbf{v}-\mathbf{U}|\big)}.$$

This leads to

$$\lambda = \mathbf{E}_{\mathbf{V}}[\, |\mathbf{V}| \times \tau(\mathbf{V})] = \frac{1}{4\pi a^2 n} \mathbf{E}_{\mathbf{X}}\Big[\frac{|\mathbf{X}|}{\mathbf{E}_{\mathbf{Y}}\big(\,|\mathbf{X}-\mathbf{Y}|\,\big)}\Big]$$

where in the last expression, $\mathbf{X}$ and $\mathbf{Y}$ are independent, standard normal random vectors. Let's focus on the denominator

$$\begin{aligned}
\mathbf{E}_{\mathbf{Y}}\big(\,|\mathbf{x}-\mathbf{Y}|\,\big) &= (2\pi)^{-3/2} \int 2\pi \sin\theta d\theta\, y^2 dy\, e^{-y^2/2} \sqrt{x^2+y^2-2xy\cos\theta} \\
&= \frac{1}{\sqrt{2\pi}} \frac{1}{3x} \int_0^\infty dy\, y e^{-y^2/2} \big[(x+y)^3 - |x-y|^3\big] \\
&= \sqrt{\frac{2}{\pi}} \Big[e^{-x^2/2} + \sqrt{\frac{\pi}{2}} \big(x+\frac{1}{x}\big) \operatorname{erf}\big(\frac{x}{\sqrt{2}}\big)\Big]
\end{aligned}$$

Then the next averaging becomes

$$\chi \equiv \mathbf{E}_{\mathbf{X}}\Big[\frac{|\mathbf{X}|}{\mathbf{E}_{\mathbf{Y}}\big(\,|\mathbf{X}-\mathbf{Y}|\,\big)}\Big] = \int_0^\infty \frac{x^3 e^{-x^2/2}\, dx}{e^{-x^2/2} + \sqrt{\frac{\pi}{2}} \operatorname{erf}\big(\frac{x}{\sqrt{2}}\big)} \approx 0.6775$$

In any case

$$\boxed{\lambda = \frac{\chi}{4\pi a^2 n}}$$

b) C_V and λ are both proportional to the number density n; assuming constant volume, this means they don't depend on temperature. $\langle v \rangle$ is proportional to $\sqrt{T}$ and therefore so is K.

c)

$$\frac{K_{CH_4}}{K_{Ar}} = \frac{f_{CH_4}}{f_{Ar}} \times \frac{a_{Ar}^2}{a_{CH_4}^2} \times \frac{\sqrt{m_{Ar}}}{\sqrt{m_{CH_4}}} = 2 \times (1.7)^{-2} \times \sqrt{2.5} \approx \boxed{1.1}$$

When counting the degrees of freedom, I have considered the methane molecule as a rigid body.

d) Since f_{CH_4} increases, the ratio will rise too.

I.4

The photoelectric effect was evidence that Planck's formula really corresponded to the energy of the light quanta/particles (the photons). This in turn, translated the wavelength spectrum of different elements to energy gaps, later to be described by quantum theories such as Bohr's.

a)

$$V = \frac{hc/e}{\lambda} - V_0$$

$$\Rightarrow h = \frac{e}{c} \frac{\Delta V}{\Delta\,(1/\lambda)} \approx \boxed{6.605 \times 10^{-34}\ J.s}$$

b)

$$W = eV_0 = \frac{hc}{\lambda} - eV = e\frac{V_2\lambda_2 - V_1\lambda_1}{\lambda_2 - \lambda_1} = \boxed{2.21\ eV}$$

c)

$$\nu_t = c\frac{V_1/\lambda_2 - V_2/\lambda_1}{V_2 - V_1} \approx \boxed{1.213 \times 10^15}$$

II.1

a)

$$\beta = \sqrt{1 - (\frac{mc^2}{E})^2} \approx 1 - \frac{m^2c^4}{2E^2} \approx 1 - 5.61 \times 10^{-7}$$

b)

$$N = N_0 \exp(-\tau/\tau_\mu) = N_0 \exp(-t/\gamma\tau_\mu) = N_0 \exp\big(-mc^2 t/E\tau_\mu\big) = N_0 \exp(-mcx/\beta E\tau_\mu)$$

therefore, the mean distance is

$$d = \frac{\beta E\tau_\mu}{m_\mu c} \approx \boxed{6.6 \times 10^5\ m}$$

II.2

$$U = -e\phi = \frac{-eq}{4\pi\varepsilon_0\sqrt{a^2 + x^2}} \approx \frac{-eq}{4\pi\varepsilon_0 a}\big(1 - \frac{x^2}{2a^2}\big)$$

comparison with standard SHO potential, $\frac{1}{2}m\omega^2 x^2$, we find

$$\boxed{\omega^2 = \frac{eq}{4\pi\varepsilon_0 a^3 m}}$$

II.3

$$V' = \frac{T'}{T}\frac{P}{P'}V \approx \boxed{105.8\ cm^3}$$

II.4

a)

$$E_n = \hbar\omega(n + \frac{1}{2}) = \hbar\omega\big(\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \cdots\big)$$

b) All of these wave functions (and none other) satisfy the Schroedinger equation and the boundary condition at infinity; but only the odd ones satisfy the boundary condition at $y = 0$. Therefore

$$E_n = \hbar\omega(2n + \frac{3}{2}) = \hbar\omega\big(\frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \cdots\big)$$

c) This is given by the squared absolute value of the overlap between the half-cut zeroth and first wave functions. The cut leads to a new normalization and an extra factor of 2.

$$P = 2\bigg|\int_0^\infty \psi_0(y)\psi_1(y)dy\bigg|^2 = \frac{4}{\pi}\bigg|\int_0^\infty e^{-y^2} y\, dy\bigg|^2 = \boxed{\frac{1}{\pi}}$$