# PSU Physics PhD Qualifying Exam Solutions
# Spring 2010

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# Candidacy Exam
# Department of Physics
# February 6, 2010
# Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.
- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.
- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants, conversions, etc.:

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|---|---|---|
| Avogadro's number | $N_A$ | $6.022 \times 10^{23}\,\mathrm{mol^{-1}}$ |
| Boltzmann's constant | $k_B$ | $1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ |
| Electron charge magnitude | $e$ | $1.602 \times 10^{-19}\,\mathrm{C}$ |
| Gas constant | $R$ | $8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$ |
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| Speed of light in vacuum | $c$ | $2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$ |
| Permittivity constant | $\epsilon_0$ | $8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$ |
| Permeability constant | $\mu_0$ | $1.257 \times 10^{-6}\,\mathrm{N\,A^{-2}}$ |
| Gravitational constant | $G$ | $6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$ |
| Standard atmospheric pressure | 1 atmosphere | $1.01 \times 10^{5}\,\mathrm{N\,m^{-2}}$ |
| Stefan-Boltzmann constant | $\sigma$ | $5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ |
| Electron rest mass | $m_e$ | $9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$ |
| Proton rest mass | $m_p$ | $1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$ |
| Origin of temperature scales | | $0\,^\circ\mathrm{C} = 273\,\mathrm{K}$ |
| 1 large calorie (as in nutrition)<br>1 inch | | $4.184\,\mathrm{kJ}$<br>$2.54\,\mathrm{cm}$ |

I–1. A ball of radius $R$ is uniformly charged with a total electric charge $Q$.

(a) Compute the electrostatic potential as a function of the radial distance from the center.

(b) Now a narrow hole is drilled in a straight line all the way across the ball going through its center. What motion is followed by an object of mass $m$ and charge $q$, with $qQ < 0$, released at rest from one end of the hole? Will the object return to its point of release; and if so, when? Neglect friction, and assume the radius of the hole is small enough not to affect the electric potential.

I–2. A particle of mass $m$ moving in one dimension is confined by a rigid wall on one side, and a harmonic force on the other. It is thus subject to a potential $V(x)$ with $V(x) = \infty$ for $x < 0$, $V(x) = \frac{1}{2}kx^2$ for $x > 0$.

Using properties of states and energy eigenvalues of the harmonic oscillator, find the quantum mechanical energy spectrum for the system given here.

I–3. In a quasi-static process $A \to B$ in which no heat is exchanged with the environment the mean pressure $\bar{p}$ of a certain gas changes with the volume according to the relation

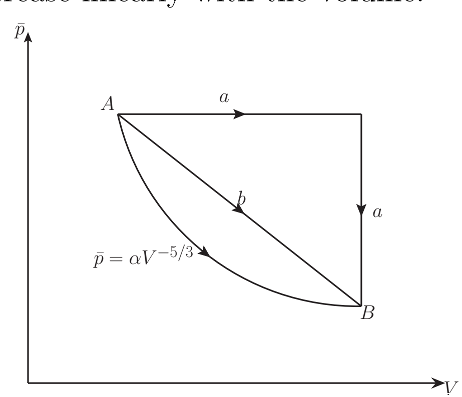
$$\bar{p} = \alpha V^{-5/3} \qquad \text{(I–1)}$$

where $\alpha$ is a constant.

Consider the same system being taken by the following two different processes from the same initial macrostate $A$ to the same final macrostate $B$. In each case, find the quasi-static work done and the heat absorbed by the system:

(a) Heat is added such that the system expands from its initial volume to its final volume at constant pressure; then heat is extracted such that the pressure decreases to its final value while the volume is kept fixed

(b) The volume is increased and simultaneously heat is supplied to cause the pressure to decrease linearly with the volume.



I–4. *Resistive magnetic monopole detector:* Maxwell's equations can be generalized to include the possibility of magnetic monopoles by adding terms for magnetic charge $Q_M$ and magnetic current $I_M$. Define the magnetic flux through a surface $S$ by $\Phi_M(S) = \int \vec{B} \cdot d\vec{S}$. Then in their integral form, the two modified Maxwell's equations are as follows: Gauss's law for $\vec{B}$ becomes

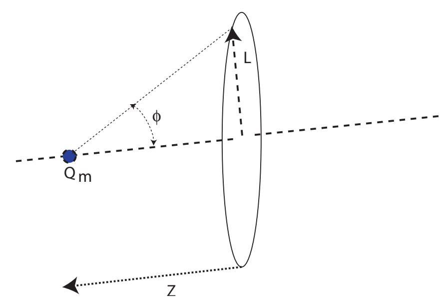
$$\Phi_M(\text{closed surface } S) = \mu_0 Q_M(\text{inside } S). \qquad \text{(I–2)}$$

The induction formula becomes

$$\oint_W \vec{E} \cdot d\vec{l} = -\frac{d\Phi_M(S)}{dt} - \mu_0 I_M(S), \qquad \text{(I–3)}$$

where $S$ is a surface with boundary $W$, and $I_M(S)$ is the magnetic current crossing $S$.

Consider the result if a particle with a magnetic charge $Q_M$ passes through a circular wire loop of radius $L$ and resistance $R$. The monopole travels with constant non-relativistic speed $v = dz/dt$ along the loop axis.




(a) Compute the flux of magnetic field through the wire loop as a function of time.

(b) Deduce the emf induced in the loop.

Note: you may find that the answer in part (a) has a sudden jump in it at the time that the particle passes through the plane of the loop. There will also be a delta function in the magnetic current at the same time, which will cancel the effects of the jump in the flux.

(c) How much heat would be dissipated in the loop as the magnetically charged particle pass through it? Assume that the energy lost to resistance is small compared to the initial kinetic energy of the particle. Ignore the inductance of the loop.

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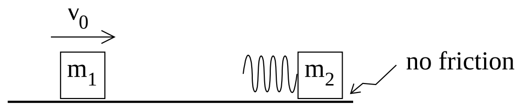
II–1. A block of mass $m_1$ move across a horizontal, frictionless table with translational velocity $v_0$, and strikes head-on a massless spring (of spring constant $k$) connected to a similar block of mass $m_2$. (See the diagram.) A maximum compression of the spring occurs at the instant the velocities of both blocks are equal. The system of the second block (mass $m_2$) and the spring are initially at rest.




If $m_1 = m_2 = 0.1\,\text{kg}$ and $k = 6.5 \times 10^4\,\text{N/m}$, what is the maximum compression amplitude $A$ of the spring when the initial translational velocity of block $m_1$ is $v_0 = 10\,\text{m/s}$?

II–2. A metal sphere of radius $a$ is placed in a uniform infinite medium of resistivity $\rho$ and relative permittivity $\epsilon_r$. At time $t = 0$ a charge $Q$ is present on the sphere. Find the potential $V(t)$ of the sphere at later times. What is the "half-life" of charge on the sphere?

Assume that the effects of self-induction are negligible.

II–3. In quantum mechanics, the one-dimensional motion of a particle of mass $m$ in a potential $V(x)$ is represented by a wave function $\psi(x, t)$.

(a) Show that the time variations of the expectation values of the position and momentum operators are given by

$$\frac{d}{dt}\langle x\rangle = \frac{\langle p\rangle}{m}, \qquad \text{and} \qquad \frac{d}{dt}\langle p\rangle = -\left\langle \frac{dV}{dx} \right\rangle. \tag{II–1}$$

(b) What do these results imply for the motion of a strongly localized wave-packet?

II–4. The water molecule has a dipole moment $\vec{p}_0$ of a fixed size. In the fluid state the dipoles point in different directions. The application of a small electric field $\vec{E}$ causes the fluid to develop a net polarization $\vec{P}$.

(a) Use classical physics to derive a formula for the polarization as a function of $p_0$, $E$, and temperature $T$, for a fluid with a density of $n$ water molecules per unit volume.

(b) Derive an expression for the dielectric function of the fluid.

## I-1

a) Inside the ball

$$\phi = \int_0^r \frac{4\pi r'^2 dr' \rho}{4\pi\varepsilon_0 r} + \int_r^R \frac{4\pi r'^2 dr' \rho}{4\pi\varepsilon_0 r'} = \frac{\rho}{2\varepsilon_0}(R^2 - r^2/3)$$

$$= \boxed{\frac{3Q}{8\pi\varepsilon_0 R^3}(R^2 - r^2/3)}$$

Outside the ball

$$\boxed{\phi = \frac{Q}{4\pi\varepsilon_0 r}}$$

b) Dropping the constant term, the potential energy inside the ball is

$$U = \frac{-qQr^2}{8\pi\varepsilon_0 R^3} = \frac{1}{2}m\frac{|qQ|}{4\pi\varepsilon_0 m R^3}r^2$$

Neglecting all radiations and frictions, this leads to a harmonic motion

$$r = R\cos\left(t\sqrt{\frac{|qQ|}{4\pi\varepsilon_0 m R^3}}\right)$$

this means that the object *will* come back to its initial position.

## I-2

The hard wall is equivalent to the boundary condition

$$\psi(0) = 0$$

which eliminates all SHO wave functions with even symmetry. The remaining states correspond to odd $n$; therefore the spectrum is

$$\boxed{E_n = \sqrt{k/m}(2n + \frac{3}{2}) \quad n = 0, 1, 2, \cdots}$$

## I-3

Define

$$r \equiv \frac{V_B}{V_A}$$

The first law of thermodynamics reads

$$U_B - U_A = W_{\text{Adiabatic}} = -\int_{V_A}^{V_B} p dV = -\alpha V_A^{-2/3}(1 - r^{-2/3})$$

a) The work is

$$W_a = -\int_{V_A}^{V_B} p dV = \boxed{-\alpha V_A^{-2/3}(r - 1)}$$

therefore the heat is

$$Q_a = W_{\text{Adiabatic}} - W_a = \boxed{\alpha V_A^{-2/3}(r + r^{-2/3} - 2)}$$

b) Similar to the previous part

$$W_b = -\int_{V_A}^{V_B} p dV = \boxed{-\frac{\alpha}{2} V_A^{-2/3}(r-1)(1+r^{-5/3})}$$

$$Q_b = W_{\text{Adiabatic}} - W_b = \boxed{\frac{\alpha}{2} V_A^{-2/3}(r - r^{-5/3} + 3r^{-2/3} - 3)}$$

# I-4

a)

$$\boxed{\Phi_M = \frac{1}{2}\mu_0 Q_M \Big( \frac{vt}{\sqrt{L^2 + v^2t^2}} - \text{sgn}(t) \Big)}$$

b)

$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_M}{dt} - \mu_0 I_M \\ &= -\frac{1}{2}\mu_0 Q_M \Big[ \frac{L^2 + v^2t^2 - v^2t^2}{(L^2 + v^2t^2)^{3/2}} v - 2\delta(t) \Big] - \mu_0 Q_M \delta(t) \\ &= \boxed{-\frac{1}{2}\mu_0 Q_M L^2 v (L^2 + v^2t^2)^{-3/2}} \end{aligned}$$

c)

$$\begin{aligned} Q &= \int \frac{\mathcal{E}^2}{R} dt = \frac{\mu_0^2 Q_M^2 v}{4LR} \int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^3} \\ &= \boxed{\frac{3\pi \mu_0 Q_M^2 v}{32LR}} \end{aligned}$$

# II-1

In the CM, the energy is

$$T = 2 \times \frac{1}{2} m (v/2)^2 = \frac{mv^2}{4}$$

at max compression

$$U = \frac{1}{2} k A^2 = \frac{mv^2}{4}$$

leading to

$$A = \sqrt{\frac{mv^2}{2k}} \approx \boxed{8.77\, mm}$$

## II-2

$$\mathbf{D} = \frac{Q(t)\hat{\mathbf{r}}}{4\pi r^2}$$

$$\mathbf{J} = \frac{\mathbf{D}}{\rho\varepsilon} = \frac{Q(t)\hat{\mathbf{r}}}{4\pi\rho\varepsilon r^2}$$

leading to

$$\dot{Q}(t) = -\frac{1}{\rho\varepsilon}Q(t) \implies Q(t) = Qe^{-t/\rho\varepsilon}$$

$$\boxed{V(t) = \frac{Qe^{-t/\rho\varepsilon}}{4\pi\varepsilon a}}$$

The half-life is

$$\boxed{\tau = \rho\varepsilon\log(2)}$$

## II-3

a)

$$\frac{d}{dt}\langle x\rangle = \frac{d}{dt}\langle\psi|\,x\,|\psi\rangle = i\,\langle\psi|\,[H,x]\,|\psi\rangle$$
$$= \frac{i}{2m}\langle[p^2,x]\rangle = \frac{\langle p\rangle}{m} \quad \blacksquare$$

$$\frac{d}{dt}\langle p\rangle = i\langle[V(x),p]\rangle = -\langle V'(x)\rangle \quad \blacksquare$$

b) The location of a strongly localized wave function in the phase space (say its Werner function), follows a classical path.

## II-4

a, b) In the presence of an external field $E\mathbf{z}$, the distribution of the dipole moment over the unit sphere will be proportional to

$$\exp(\beta p_0 E\cos\theta)$$

therefore

$$\langle\cos\theta\rangle = \frac{\int_0^\pi d\theta\sin\theta\cos\theta e^{\beta p_0 E\cos\theta}}{\int_0^\pi d\theta\sin\theta e^{\beta p_0 E\cos\theta}}$$
$$= \coth(\beta p_0 E) - \frac{1}{\beta p_0 E}$$

Therefore

$$\mathbf{P} = np_0\langle\cos\theta\rangle\hat{\mathbf{E}}$$

Which gives

$$\boxed{\chi_e = \frac{\beta n p_0^2}{\varepsilon_0}g(x)}$$

where

$$x \equiv \beta p_0 E \quad ; \quad g(x) = \frac{\coth(x)}{x} - \frac{1}{x^2}$$

for small fields, this is

$$\chi_e = \frac{\beta n p_0^2}{3\varepsilon_0}$$