PSU Physics PhD Qualifying Exam Solutions
Spring 2016

Koorosh Sadri

September 16, 2023

Qualifying Exam for Ph.D. Candidacy
Department of Physics
February 6th, 2016

Part I

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.

- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.

- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants, conversions, etc.:

Avogadro's number	N_A	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Electron charge magnitude	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Gas constant	R	$8.314\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Planck's constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34}\,\mathrm{J\,s}$
Speed of light in vacuum	c	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability constant	μ_0	$1.257 \times 10^{-6}\,\mathrm{N\,A^{-2}}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5}\,\mathrm{N\,m^{-2}}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Electron rest mass	m_e	$9.109 \times 10^{-31}\,\mathrm{kg} = 0.5110\,\mathrm{MeV}\,c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27}\,\mathrm{kg} = 938.3\,\mathrm{MeV}\,c^{-2}$
Origin of temperature scales		$0\,^\circ\mathrm{C} = 273\,\mathrm{K}$
1 large calorie (as in nutrition)		$4.184\,\mathrm{kJ}$
1 inch		$2.54\,\mathrm{cm}$

Definite integrals:

$$\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$ (I–1)

$$\int_0^\infty x^n e^{-x} dx = \Gamma(n+1) = n!.$$ (I–2)

I–1. A neutral K^0 meson decays into two charged pions

$$K^0 \to \pi^+ + \pi^- .$$

The pions have opposite charges, equal in magnitude to the electron charge $e = 1.602 \times 10^{-19} C$. The rest masses of pions are equal to $m_\pi = 140 MeV/c^2$, where the unit of energy MeV (mega-electronvolt) is equal to $1 MeV = 1.602 \times 10^{-13} J$ and $c = 3 \times 10^8 \, m/s$ is the speed of light.
Assume that the K^0 meson is initially at rest in the bubble chamber with the magnetic field $B = 2 \, T$ and it decays to the pions. Assume furthermore that pions have momenta perpendicular to the magnetic field. The radius of curvature of the pions is measured to be equal to $R = 34.4 \, cm$.

(a) Determine the value of the momentum of each pion (express it in units of MeV/c).

(b) Find the speed of the pions as a fraction of the speed of light c.

(c) Find the energy of each pion (in units of MeV).

(d) Determine the mass of the meson K^0 (in units of MeV/c^2)

I–2. A particle in a central potential has an orbital angular momentum $l = 2\hbar$ and a spin angular momentum $s = 1\hbar$. Find the energy levels and degeneracies associated with a spin-orbit interaction term of the form

$$H_{so} = A \, \vec{L} \cdot \vec{S} ,$$

where A is a constant and $\vec{L}$ and $\vec{S}$ denote orbital and spin angular momenta, respectively.

I–3. Consider a point charge q positioned a distance d away from the center of an *ungrounded* conducting sphere which carries a net charge Q. Assume the charges q and Q have the same sign. Using the method of images with two image charges, determine the force on the point charge q.
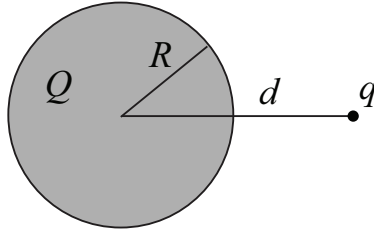

Figure I–3: Conducting sphere and point charge.

I–4. A mixture of ℓ perfect gases, consisting of equal numbers of particles with different masses $m_1, \ldots, m_\ell$, is contained in a cylinder of radius R and height h and is placed in the Earth's gravitational field. Assuming that the height of the cylinder is much smaller than the radius of the Earth, determine the center of gravity of the mixture.

Qualifying Exam for Ph.D. Candidacy
Department of Physics
February 6th, 2016

Part II

Instructions:

- The following problems are intended to probe your understanding of basic physical principles. When answering each question, indicate the principles being applied and any approximations required to arrive at your solution. *If information you need is not given, you may define a variable or make a reasonable physical estimate, as appropriate.* Your solutions will be evaluated based on clarity of physical reasoning, clarity of presentation, and accuracy.

- Please use a new blue book for each question. Remember to write your name and the problem number of the cover of each book.

- We suggest you read all *four* of the problems before beginning to work them. You should reserve time to attempt every problem.

Fundamental constants, conversions, etc.:

Avogadro's number	N_A	$6.022 \times 10^{23} \, \text{mol}^{-1}$
Boltzmann's constant	k_B	$1.381 \times 10^{-23} \, \text{J} \, \text{K}^{-1}$
Electron charge magnitude	e	$1.602 \times 10^{-19} \, \text{C}$
Gas constant	R	$8.314 \, \text{J} \, \text{mol}^{-1} \, \text{K}^{-1}$
Planck's constant	h	$6.626 \times 10^{-34} \, \text{J} \, \text{s}$
	$\hbar = h/2\pi$	$1.055 \times 10^{-34} \, \text{J} \, \text{s}$
Speed of light in vacuum	c	$2.998 \times 10^{8} \, \text{m} \, \text{s}^{-1}$
Permittivity constant	ϵ_0	$8.854 \times 10^{-12} \, \text{F} \, \text{m}^{-1}$
Permeability constant	μ_0	$1.257 \times 10^{-6} \, \text{N} \, \text{A}^{-2}$
Gravitational constant	G	$6.674 \times 10^{-11} \, \text{m}^3 \, \text{kg}^{-1} \, \text{s}^{-2}$
Standard atmospheric pressure	1 atmosphere	$1.01 \times 10^{5} \, \text{N} \, \text{m}^{-2}$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8} \, \text{W} \, \text{m}^{-2} \, \text{K}^{-4}$
Electron rest mass	m_e	$9.109 \times 10^{-31} \, \text{kg} = 0.5110 \, \text{MeV} \, c^{-2}$
Proton rest mass	m_p	$1.673 \times 10^{-27} \, \text{kg} = 938.3 \, \text{MeV} \, c^{-2}$
Origin of temperature scales		$0 \, °\text{C} = 273 \, \text{K}$
1 large calorie (as in nutrition)		$4.184 \, \text{kJ}$
1 inch		$2.54 \, \text{cm}$

Definite integrals:

$$\int_0^\infty e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}. \tag{II–1}$$

$$\int_0^\infty x^n e^{-x}\,dx = \Gamma(n+1) = n!. \tag{II–2}$$

II–1. The point of support of a simple pendulum of length b moves on a massless rim of radius a rotating with constant angular velocity ω (see Figure II-1). Obtain the expression for the Cartesian components of the velocity and acceleration of the mass m. Obtain also the angular acceleration for the angle θ shown in Figure II-1.
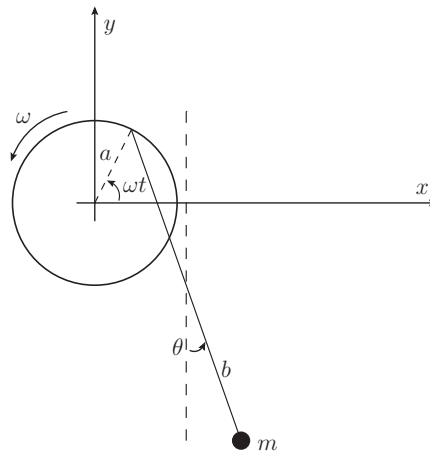

Figure II–1: Pendulum on rotating rim.

II–2. The Hamiltonian for a spin-1/2 particle with charge q in an external magnetic field $\vec{B}$ is

$$H = -\frac{gq}{2mc}\vec{S}\cdot\vec{B}\ ,$$

where $\vec{S}$ is the spin angular momentum and g is the gyromagnetic ratio. Calculate the operator $\frac{d\vec{S}}{dt}$ if $\vec{B} = B\hat{e}_y$. What is $S_z(t)$ in matrix form?

II–3. A square loop of wire of side a lies on a table, a distance s from a very long straight wire which carries a current I, as shown in Figure II-3.

(i) Find the flux of the magnetic field through the loop

(ii) If someone now pulls the loop directly away from the wire, at speed v, what emf is generated? In what direction does the induced current flow?

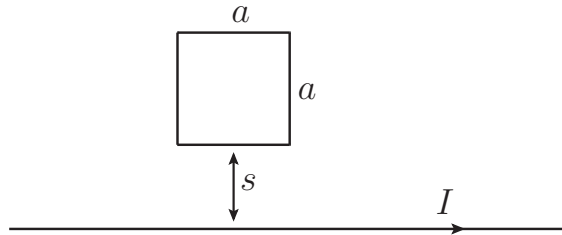(iii) What emf is generated if the loop is pulled to the right at speed v?

Figure II–3: Wire and square loop

II–4. A material is brought from temperature T_i to T_f by placing it in contact with a series of N reservoirs of temperatures $T_i + \Delta T$, $T_i + 2\Delta T, \ldots, T_i + N\Delta T = T_f$. Assuming that the heat capacity C of the material is temperature independent, calculate the entropy change of the total system, material plus reservoirs. What is the entropy change in the limit $N \to \infty$ for fixed $T_f - T_i$.

I-1

a) The formula for the radius is

$$R = \frac{p}{Bq}$$

therefore

$$p = 206.4 \ MeV/c^2$$

b)

$$\beta = \frac{p}{\sqrt{p^2 + m^2c^2}} \approx 0.828$$

c)

$$E = \sqrt{m^2c^4 + p^2c^2} \approx 249.4 \ MeV$$

d)

$$M_{K^0} = 2E/c^2 \approx 498.8 \ MeV$$

I-2

$$\frac{A}{2}(\mathbf{L} + \mathbf{S})^2 = \frac{A}{2}(L^2 + S^2 + 2\mathbf{S.L}) = 4A\hbar^2 + H$$

Therefore

$$H = \frac{A}{2}(\mathbf{L} + \mathbf{S})^2 - 4A\hbar^2$$

Therefore

$$E_j = \frac{1}{2}A\hbar^2[j(j+1) - 8] \ ; \ \ j \in \{1, 2, 3\} \ ; \ \ \Omega_j = 2j + 1$$

I-3

$$\mathbf{F} = \frac{q}{4\pi\varepsilon_0} \left(\frac{Q}{d^2} + \frac{Rq}{d^3} - \frac{Rdq}{(d^2 - R^2)^2} \right) \hat{\mathbf{r}}$$

I-4

Assuming that the temperature is constant throughout the height of the cylinder, the constant gravity assumption leads to an exponential decay of the density of each gas with the height. The decay constant being

$$h_i = \frac{k_B T}{m_i g}$$

Therefore, each gas has its center of mass at a height

$$z_i = \frac{\int_0^h dz \, z e^{-z/h_i}}{\int_0^h dz \, e^{-z/h_i}} = h_i \left(1 - \frac{h/h_i}{e^{h/h_i} - 1} \right)$$

Therefore the center of mass of the mixture is

$$z = \frac{\sum_i N m_i z_i}{\sum_i N m_i} = \frac{\sum_i m_i z_i}{\sum_i m_i}$$

$$= \frac{k_B T}{g \sum_i m_i} \sum_i \left(1 - \frac{h/h_i}{e^{h/h_i} - 1} \right)$$

II-1

$$x = a \cos \omega t + b \sin \theta \quad ; \quad y = a \sin \omega t - b \cos \theta$$
$$\dot{x} = -\omega a \sin \omega t + b \dot{\theta} \cos \theta$$
$$\dot{y} = \omega a \cos \omega t + b \dot{\theta} \sin \theta$$
$$\ddot{x} = -\omega^2 a \cos \omega t - b \dot{\theta}^2 \sin \theta + b \ddot{\theta} \cos \theta$$
$$\ddot{y} = -\omega^2 a \sin \omega t + b \dot{\theta}^2 \cos \theta + b \ddot{\theta} \sin \theta$$

The Lagrangian is equivalent to

$$L = \frac{1}{2} m b^2 \dot{\theta}^2 + mab\omega \dot{\theta} \sin(\theta - \omega t) + mgb \cos \theta$$

leading to

$$\ddot{\theta} = \frac{a}{b} \omega^2 \cos(\theta - \omega t) - \frac{g}{b} \sin \theta$$

II-2

$$\frac{d\mathbf{S}}{dt} = i[H, \mathbf{S}] = \frac{-igqB}{2mc}[S_y, \mathbf{S}] = \frac{gqB\hbar}{2mc}(-S_z, 0, S_x)$$

Solved for example, as

$$S_z(t) = \cos\left(\frac{gqB\hbar t}{2mc}\right) S_z(0) + \sin\left(\frac{gqB\hbar t}{2mc}\right) S_x(0) = \frac{\hbar}{2} \begin{pmatrix} \cos(gqB\hbar t/2mc) & \sin(gqB\hbar t/2mc) \\ \sin(gqB\hbar t/2mc) & -\cos(gqB\hbar t/2mc) \end{pmatrix}$$

II-3

i)

$$\Phi = \frac{\mu_0 I}{2\pi} \int_s^{s+a} \frac{a\,dr}{r} = \frac{\mu_0 I a}{2\pi} \log\left(1 + \frac{a}{s}\right)$$

ii)

$$\mathcal{E} = \frac{\mu_0 I a^2 v}{2\pi s(a+s)}$$

The current is CCW.

iii) No emf.

II-4

$$\Delta S_{tot.} = C \sum_{n=0}^{N-1} \left[\log\left(1 + \frac{\Delta T}{T_i + n\Delta T}\right) - \frac{\Delta T}{T_i + (n+1)\Delta T} \right]$$

In the large N limit:

$$\Delta S_{tot.} \approx \frac{C(T_f - T_i)^2}{2N^2} \sum_{n=0}^{N-1} \frac{1}{[T_i + n(T_f - T_i)/N]^2} \to 0$$